

## Problem 2.40

[Difficulty: 2]

**2.40** The velocity distribution for laminar flow between parallel plates is given by

$$\frac{u}{u_{\max}} = 1 - \left(\frac{2y}{h}\right)^2$$

where  $h$  is the distance separating the plates and the origin is placed midway between the plates. Consider a flow of water at  $15^\circ\text{C}$ , with  $u_{\max} = 0.10 \text{ m/s}$  and  $h = 0.1 \text{ mm}$ . Calculate the shear stress on the upper plate and give its direction. Sketch the variation of shear stress across the channel.

**Given:** Velocity distribution between flat plates

**Find:** Shear stress on upper plate; Sketch stress distribution

**Solution:**

Basic equation  $\tau_{yx} = \mu \frac{du}{dy}$   $\frac{du}{dy} = \frac{d}{dy} u_{\max} \left[ 1 - \left( \frac{2y}{h} \right)^2 \right] = u_{\max} \left( -\frac{4}{h^2} \right) \cdot 2y = -\frac{8 \cdot u_{\max} \cdot y}{h^2}$

$$\tau_{yx} = -\frac{8 \cdot \mu \cdot u_{\max} \cdot y}{h^2}$$

At the upper surface  $y = \frac{h}{2}$  and  $h = 0.1 \text{ mm}$   $u_{\max} = 0.1 \frac{\text{m}}{\text{s}}$   $\mu = 1.14 \times 10^{-3} \frac{\text{N} \cdot \text{s}}{\text{m}^2}$  (Table A.8)

Hence  $\tau_{yx} = -8 \times 1.14 \times 10^{-3} \frac{\text{N} \cdot \text{s}}{\text{m}^2} \times 0.1 \frac{\text{m}}{\text{s}} \times \frac{0.1}{2} \text{ mm} \times \frac{1 \cdot \text{m}}{1000 \cdot \text{mm}} \times \left( \frac{1}{0.1 \cdot \text{mm}} \times \frac{1000 \cdot \text{mm}}{1 \cdot \text{m}} \right)^2$   $\tau_{yx} = -4.56 \frac{\text{N}}{\text{m}^2}$

The upper plate is a minus  $y$  surface. Since  $\tau_{yx} < 0$ , the shear stress on the upper plate must act in the plus  $x$  direction.

The shear stress varies linearly with  $y$   $\tau_{yx}(y) = -\left( \frac{8 \cdot \mu \cdot u_{\max}}{h^2} \right) \cdot y$

